Faculty Working Papers

THE THEORETICAL STRUCTURE OF AUDIT OPINION SPACE

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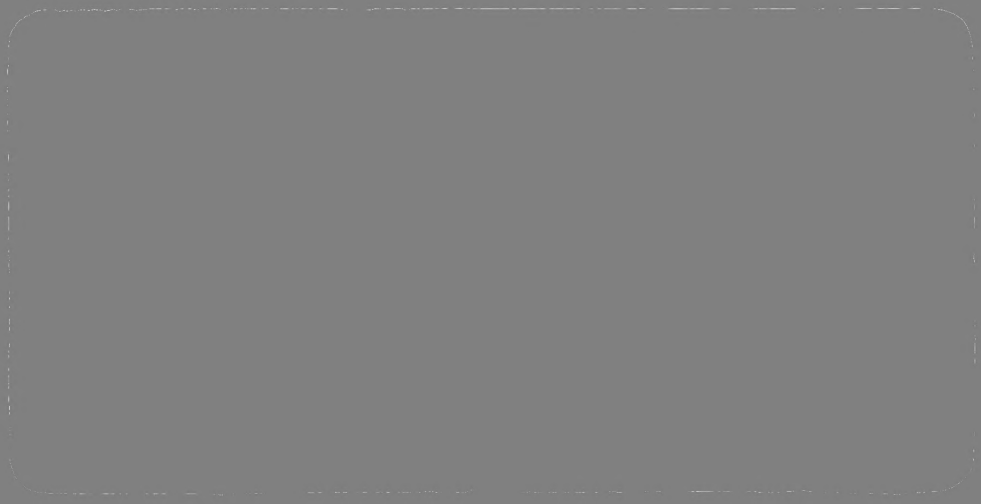

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by

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THE THEORETICAL STRUCTURE OF AUDIT OPINION SPACE

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Auditing is essentially a decision process, ultimately reaching an auditor's opinion as to the overall fairness of the financial reporting. Currently, there are four opinion alternatives available to the auditor, one of which he must choose at the end of his examination. These opinions were promulgated by the American Institute of CPAs (AICPA) in the form of a descriptive model and are well accepted by the public and the S.E.C.

Various attempts have been made to use decision science techniques to improve the audit process. Arkin [3], Neter [14], Elliott and Rogers [8], and Roberts [15] were concerned with the usefulness of the classical statistical sampling techniques. Birnberg [4], Sorensen [17], Tracy [19], and Corless [6] tried to apply Bayesian sampling techniques. More recently, Ijiri and Kaplan [11] attempted to use a goal programming approach to satisfy the multiple goals of an auditor's sampling, while Yu and Neter [20] used a stochastic concept to evaluate the reliability of the internal control system. On the other hand Davis [7] explored the usefulness of the network technique in planning an audit program.

All of this research was concerned with more efficient audit processes in generating evidence. Little research has been done on how the evidence and information so obtained can be used to reach an audit opinion decision.

The purpose of this paper is to propose a model that will reveal the relationship between the audit opinion parameters or factors that affect an audit opinion and the AICPA's four audit opinion alternatives. The opinion parameters are first identified and an attempt is then made to define the parameters in quantitative terms. The quantified parameters are then considered

basis vectors generating an audit opinion space which is partitioned into four mutually exclusive subspaces, each representing one of the opinion alternatives. The auditor's utility function is related to the determination of the partitioning criteria of the audit opinion space. The resulting model expressed in mathematical terms is based on the AICPA's descriptive model set forth in the Statement on Audit Standards No. 1 [2].

The significance of this study is threefold: (1) it develops a means to quantify various dimensions of an item that is considered significant for the audit purpose. The quantified measures of an item in multiple dimensional terms would enable the auditor to systematically handle the effect of the various significant items in audit, (2) it attempts for the first time to provide a basis for a theoretical explanation of the profession's practices with regard to the audit opinion decision, and (3) it may provide a useful step toward a formal decision-theoretic approach in auditing.

Alternatives of Audit Opinions

According to the Statement on Auditing Standards No. 1 of the AICPA, there are four opinion alternatives available to an auditor, one of which must be issued at the conclusion of an audit. They are the unqualified opinion, the qualified opinion, the adverse opinion, and the disclaimer of opinion. Although the meaning of these different alternatives are well known, the basic definitions are given below because they are the models on which theory of this paper is constructed.

Unqualified opinion. This opinion, sometimes referred to as a "clean opinion," is issued on the financial statements when the independent auditor has formed the opinion, that on the basis of an examination made in accordance with generally accepted auditing standards, the presentation of the financial

statements, taken as a whole, is fair, in conformity with generally accepted accounting principles applied on a consistent basis and includes all informative disclosures necessary to make the statements not misleading.

Qualified opinion. This opinion modifies the unqualified opinion by stating that the financial statements, taken as a whole, are presented fairly with certain exception, the effects of which are material enough that they may not be ignored. The exceptions may arise due to departures from either generally accepted auditing standards or generally accepted accounting principles, or due to inconsistent application of accounting principles, inadequate disclosure of information, or uncertainty as to the state of nature. The effect of the exception, though material, can be localized in its relation to the other parts of the financial statements so as not to impair the overall fairness.

Adverse opinion. This is the opposite of an unqualified opinion: it is an opinion that the financial statements do not present fairly the financial position, results of operations, or changes in financial position in conformity with generally accepted accounting principles. This opinion is required in any auditor's report when exceptions are so significantly material that, in the auditor's judgment, a qualified opinion is not justified; that is, the effects of the exceptions are so pervasive that the overall fairness of the financial statements, taken as a whole, is definitely destroyed.

Disclaimer of opinion. When the auditor was unable to accumulate sufficient competent evidential matter to form an opinion on the financial statements, he should so state through a disclaimer of opinion. The necessity of disclaiming an opinion may arise either from a serious limitation on the scope of examination or from the existence of unusual uncertainties concerning the amount of an item or the outcomes of a matter materially affecting financial

statements causing the auditor not to be able to form an opinion on the statements taken as a whole.*

The four alternatives promulgated by the AICPA are in the form of a descriptive model, focusing on the concept of fair presentation of financial statements. In very simplistic terms, the four opinions may be stated as follows: An unqualified opinion is issued when the auditor is convinced, upon completing an audit in accordance with generally accepted auditing standards, that the financial statements are fairly presented; on the other hand, an adverse opinion is issued when the auditor is convinced that the statements are not presented fairly. A disclaimed opinion is issued when the auditor has little or no basis to judge the fairness of the financial statement due to his ignorance (or uncertainty) as to the states of nature surrounding the financial statements. The qualified opinion, which is basically a modified unqualified opinion having an element of unfairness or uncertainty in the financial statements, falls somewhere between these three opinions.

Despite the fact that the concept of fair presentation of the financial statements is the focal point of the auditor's opinion, the literature does not provide a clear-cut meaning of fair presentation. The AICPA defines the concept indirectly by listing five criteria circumstances where fair presentation of the financial statements is deemed impaired. In the absence of the circumstances the auditor may conclude that the financial statements under audit are fairly presented.

The five circumstances are:

1. The scope of the auditor's examination is limited or affected;

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*For a detailed discussion of these four opinions, see the AICPA [2], pp. 80-83.

2. Unusual uncertainties exist concerning future developments, the effect of which cannot be reasonably estimated or otherwise resolved satisfactorily;

3. The financial statements are not presented in conformity with generally accepted accounting principles;

4. Accounting principles are not consistently applied;

5. Informative disclosure of qualitative and quantitative information is not adequately made.

When an auditor finds a significant item or event during his examination, his mind undergoes a quick thought process,* such as running through a decision table, as follows: Is this item in accordance with generally accepted accounting principles; is the item treated consistently; should the item be disclosed; is all necessary evidence related to the item obtainable without the client's interference; and is there any unusual uncertainty related to the item? These questions clearly imply that any significant item or event does potentially have five aspects corresponding to the five circumstances listed above. Here, we make three observations: (1) the item or event may be subject to more than one circumstance, (2) each circumstance may have a varying degree of seriousness, and (3) there could be many significant items or events in a financial statement that need scrutiny simultaneously from the view of the five circumstances. These observations, though they may appear trivial, provide a set of bases for the development of the theory in this paper.

Strictly speaking, all assertions in the financial statements and the related events thereon should be carefully examined for their potential sensitivity to the five criteria circumstances. But, for all practical purposes, most of the assertions and events are verified beyond reasonable doubt as to their conformity with the conditions supporting the "unqualified"

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*The AICPA [1], pp. 98-123.

opinion. Only a relatively limited number of assertions and events are potential candidates as those significant items that may force the auditor to consider other than an unqualified opinion. In this paper, we define E_i, i=1, 2, ···, n, as the potential candidates for significant items, and show how they influence the choice of the auditor's opinion alternatives.

A question that immediately arises is whether the auditor should always issue an opinion other than unqualified whenever one or more of the circumstances are present in any E_i. The literature suggests that the answer depends on the seriousness of E_i's. That is, the audit opinion to be issued must be determined by considering the interface of the set of significant events E_i, i=1, 2, ···, n, having varying degrees of seriousness in terms of the five criteria circumstances, with the auditor's criteria functions, which presumably reflect his utility function.

Model of the Auditor's Opinion Space

We now construct the auditor's opinion space, in which the interface can be depicted. First, we define three distinctive parameters as inherent elements in the five circumstances, eac parameter representing a basis vector. Secondly, each specific event E_i will be measured in terms of the three parameters, just as in geometry a point in space is measured in terms of three axes, X, Y, and Z. Since auditors currently do not have quantified measurement units with respect to the parameters, we define the units in quantitative terms possessing a certain desirable property.

Our objective is to provide a means to express various events E_i in common denominators so that they can be systematically related to the audit opinion decision criteria function.

Parameter 1: Uncertainty as to the State of Nature. Auditing is an

attestation to the credibility of management's assertions embodied in the financial statements. The assertions are management's claims concerning the state of nature that prevails with respect to the company's financial position, results of operations, and changes in financial position. The credibility of an assertion should be substantiated by evidence collected and evaluated by the auditor. If the auditor cannot obtain enough evidence, he is necessarily uncertain and uncomfortable because he has less than convincing knowledge as to the state of nature surrounding those assertions. The auditor's immediate goal is to obtain as much evidence as practical to decrease the degree of uncertainty to acceptable bounds. But as in any other field that is concerned with evidence, auditing is subject to various limitations in its effort to collect evidence. One limitation is the audit scope imposed by the management or by factors like cost, time, or other conditions which preclude the use of auditing procedures considered necessary in the circumstances. Another limitation the auditor faces is temporal in nature. To the extent accounting principles rely on the going-concern assumption, the auditor has to make some prognostication as to future events and outcomes on the basis of evidence now available to him. Uncertainties concerning future developments necessarily leave the auditor in a certain stage of ignorance as to the state of nature surrounding the financial statements. Pending law suits or exact bad debt amounts are typical examples. An examination of the five circumstances that require the auditor to depart from the unqualified opinion suggests that the first two--limitation on audit scope and uncertainties about future developments--may now be summarized and represent but one parameter: the degree of uncertainty as to the state of nature associated with the financial report.

The degree of uncertainty is inversely related to the degree of sufficiency

and competency of the evidence. The more sufficient and competent evidence the auditor has, the less uncertain he can be. For example, assume that the auditor finds a significant item E_i in the financial statements. The auditor then obtains evidence to determine the true state of nature with respect to E_i. What the evidence reveals, however, may or may not correspond to the true state of nature. In general, the reliability of evidence is at best probabilistic. Under the current state of art in auditing whether the evidence is sufficient and competent enough to reveal the true state of nature is, in most cases, a matter of professional judgment. This professional judgment may be expressed in terms of the auditor's subjective probability.* Let such a probability be denoted as $p(E_i)$. Then a measure of the uncertainty, denoted as U, associated with the evidence with respect to E_i, may be defined as:

$$U = \log \frac{1}{p(E_i)}, \qquad (1)$$

which increases from 0 to ∞ as $p(E_i)$ decreases from 1 to 0. This is a monotone function. Notice that it is the measure of information in communication theory [16]. This measure is consistent with the theory of audit evidence: when the auditor has less evidence, his perception as to the state of nature should be more uncertain--this inverse relation is represented by equation (1). The logarithmic function allows us to easily manipulate U, which will be shown later.

Parameter 2: Monetary Effects. Most of the exceptional circumstances that concern the auditor can be measured in monetary terms. The two conspicuous circumstances identified by the AICPA are the departure from generally accepted accounting principles and the violation of the principle of consistency. Since

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*The subjective probability may vary from auditor to auditor. But the AICPA's "generally accepted auditing standards" provide a set of criteria which, if followed, helps minimize divergence of such probability. The subjectivity of the probability does not hinder the model development. Our concern is the quantification of such subjective assessment of the quality of evidence.

the effects of these events are measured in monetary terms, the two circumstances may be treated under one common denominator: the monetary effect. This second parameter can also accommodate other unusual events such as contingent liabilities, the effect of merging under a "pooling of interests," and the effect of nationalization of a subsidiary by a foreign government. The auditing problem is to determine how significant the monetary effects of these unusual events should be before the auditor seriously considers choosing other opinions. It is the problem of "materiality." Despite the daily use of the concept of materiality in every audit, auditors are devoid of a reliable measure that can be used as a reference in determining whether an item is material. In his empirical study Frishkoff [9] identified two materiality criteria variables that significantly influence audit opinions: one is the ratio of an item to net income, and the other is the ratio of an item to the "net worth." One way of combining these two variables is to create an index M as follows:

$$M = v_1 \left|\frac{e_i}{NI}\right| + v_2 \left|\frac{e_i}{NW}\right|, \qquad (2)$$

where e_i is the monetary effect of event E_i in question, NI represents net income ($NI \neq 0$), NW represents the net worth ($NW \neq 0$), and v_1 and v_2 represent relative weights satisfying the conditions $v_1 > 0$, $v_2 > 0$, and $v_1 + v_2 = 1$. The measure M is always positive and is a monotone function; its magnitude reflects the degree of seriousness of the exceptional item in relation to the net income and size of the company. This approach avoids Hick's criticism [10] of using the net income as a sole materiality criterion: M is less volatile because of its association with a stable basis of the net worth. Moreover, once M is obtained, it becomes a relative measure independent of the size of the company.*

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*We may add other criteria variables in measuring M. See [21] for other variables. Which criteria variables and what relative weights should be used is not a main concern in this paper. They shall be determined by individual auditors. What is concerned is that M be an index, measuring consistently and systematically the degree of materiality.

Parameter 3: Disclosure. The last circumstance the AICPA identifies as a condition under which the auditor may have to choose other than an unqualified opinion relates to the adequacy of informative disclosure. Recognizing that not all financial information can be reduced to a set of numbers in financial statements, additional information, both qualitative and quantitative, needs to be supplied in order not to mislead financial statements users. Auditors do not have any systematic means of measuring the degree of adequacy of disclosure.

Assuming the objective of disclosure is to disseminate information so that investors will not be misled, the quality of a disclosure may be expressed in terms of its ability to reduce the probability of misleading investors. This leads to the idea of information theory used by Theil [18]. Assume an event E_i that may need disclosure to prevent investors from being misled. Assume also that the auditor believes the probability of misleading the investors without disclosure of E_i is $r(E_i)$, and the probability of misleading the investors with disclosure of E_i is $q(E_i)$. Then by definition, disclosure should occur when $r(E_i) \geq q(E_i)$. The information content associated with event E_i without disclosure of the event is then measured (according to information theory) as $\log (1/r(E_i))$; and the information content associated with event E_i with its disclosure is $\log (1/q(E_i))$. Given the additivity of the information concept, an obvious way of defining the quantity of information associated with the particular disclosure on E_i is:

$$\log \frac{1}{r(E_i)} - \log \frac{1}{q(E_i)} = \log \frac{q(E_i)}{r(E_i)} \qquad (3)$$

This is a measure showing how much the disclosure helps reduce the probability of misleading. It is always non-positive for $r(E_i) \geq q(E_i)$: the negative sign is understandable because disclosure with $q(E_i)$ contributes negatively to the chance of misleading. But the disclosure which helps reduce the probabilities of misleading from $r(E_i)$ to $q(E_i)$ gives no definite clue as to the question of

whether the disclosure will ultimately mislead investors. If investors are ultimately misled, the information value of the disclosure is log $(q(E_i)/r(E_i))$. The disclosure states that the chance of misleading, and hence also the chance of log $(q(E_i)/r(E_i))$ bits of information, is $q(E_i)$. Therefore, we can obtain the expected information of the disclosure which transforms the probabilities of misleading from $r(E_i)$ to $q(E_i)$ as follows:

$$D(q(E_i):r(E_i)) = q(E_i) \log \frac{q(E_i)}{r(E_i)} + (1 - q(E_i)) \log \frac{1-q(E_i)}{1-r(E_i)} \qquad (4)$$

Notice that D which is a monotone function is always positive except for the case where $r(E_i) = q(E_i)$, in which case $D = 0$.* This is consistent with the idea of disclosure because if a disclosure does not help reduce the chance of misleading the value of the disclosure is naturally nil. But, in general, disclosure, by definition, has some positive value.

Case of Multiple Events

To this point we have identified three distinctive parameters affecting singly or in combination the auditor's opinion and we have defined them in quantitative terms; all of these quantitative terms take values ranging from 0 to ∞.

The definitions of U, M, and D so far have all been made on a single event, E_i. But audits of many ordinary financial statements encounter a number of unusual events, each subject to different quantities of U, M, and D. The auditor must consider all of these items of differing levels of importance and decide upon the most desirable opinion. A model of audit opinion space should then presuppose a means of aggregating in a systematic and meaningful way the different parameter values of different events E_i, $i=1, 2, \cdots, n$, in such a way that the relative importance of E_i is properly taken care of.

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*For a more general discussion, see H. Theil [18], pp. 460-461 and B. Lev [12], pp. 18-20.

One way of aggregating the effects of individual events is a linear combination. By design, the parameters U, M, and D have been constructed so as to permit the use of a linear combination. Let the events the auditor considers unusual be $E_1,\ldots, E_i\ldots, E_n$; without losing generality, it may be assumed they are independent events. Let the monetary effects associated with these unusual events be $e_1,\ldots, e_i,\ldots, e_n$, respectively, and let the relative weight of e_i be w_i, where

$$w_i = \frac{|e_i|}{\Sigma_i^n |e_i|} \quad , \qquad i = 1, 2,\ldots, n. \tag{5}$$

The combined effect of $e_1,\cdots, e_n$ on M, using the property of the linear aggregation of monetary information in accounting, can be expressed from formula (2) as follows:

$$M = v_1 \frac{\Sigma|e_i|}{|NI|} + v_2 \frac{\Sigma|e_i|}{|NW|} \tag{6}$$

The combined effects of $e_1,\cdots, e_n$ on U and D are, from formulas (1) and (4) and the independence assumption, expressed as follows:

$$U = \Sigma w_i \log \frac{1}{p(E_i)} \quad , \tag{7}$$

$$D = \Sigma w_i\, q(E_i) \log \frac{q(E_i)}{r(E_i)} + \Sigma w_i\, (1-q(E_i)) \log \frac{1-q(E_i)}{1-r(E_i)}. \tag{8}$$

This linear combination is, of course, justified by the additivity property of the information concept mentioned earlier. The use of the monetary effect e_i as the means of determining the relative weight of event E_i and hence determining D and U, may be subject to some criticism. If there is a very important qualitative event that may have negligible immediate monetary effect (negligible e_i), such an event, under equation (8), has little effect on the measurement of D, even if the important event is well disclosed. If that is the case, the index D is not a proper measure of the quality of disclosure. This criticism is understandable. However, the truth of the matter is, that

if the auditor genuinely believes the monetary effect is negligible, he should not be bothered with the disclosure of the event. The fact that he considers the event important (even if its immediate monetary effect is negligible) signifies the auditor's perception that the potential monetary effect is material. The inference is that the auditor should always try to estimate the potential monetary effect. Whether the estimation is reliable is irrelevant as far as the determination of M is concerned. The reliability of such estimation is independently taken care of by the first parameter U.

So far we have recognized the three attributes of event E_i and defined them as three audit opinion parameters. Also we have discussed ways of expressing them in quantitative terms i.e., U, M, and D. Thus, we have provided three basis vectors to define a three dimensional audit opinion space. Furthermore, we have considered means to combine the effects of multiple events $E_1, E_2, \cdots, E_n$ on U, M, and D. In any given audit engagement, the auditor computes for each E_i, i=1, 2, $\cdots$, n, three numbers corresponding to three parameters and combine the respective numbers to come out with overall indexes U, M, and D. This forms a vector.

Our next task is to relate the four audit opinion alternatives to this three dimensional space. The motivation for this effort is twofold: One is to show the interrelationships between the parameters, the opinion alternatives, and the auditor's utility function. Specifically, we will partition the space into four subsets corresponding to the four audit alternatives. Another motivation is to show how a specific vector obtained in an audit engagement can be used in determining an audit opinion.

Partition of Parameter Space

U, M, and D defined above are, in fact, continuous variables having the

range [0, ∞). We can conceive of each variable as having a range which is partitioned into four subsets, representing the unqualified opinion, qualified opinion, adverse opinion, and disclaimer of opinion, respectively. Let u_1, u_2, u_3, u_4 represent subset intervals of U, each denoting a set that correspond to one of four opinions. These subsets are mutually exclusive and collectively exhaustive when the other two conditions, M and D, are given. This uniqueness condition of each subset is obvious because no one point should be classified in more than one opinion's subsets when the other conditions are held constant. The same concept is applicable to M and D. From this analysis we propose the following basic statement:

> Statement 1. Let u_i, m_i, d_i ($i = 1, 2, 3, 4$, each representing one of the four opinions) be subsets of U, M, and D respectively, where U, M, and D are audit parameters as discussed earlier. Then:
>
> (a) Given $m \not\equiv M$ and $d \not\equiv D$, U may be partitioned so that:
> $$\bigcup_{i=1}^{4} u_i = T, \qquad \bigcap_{i=1}^{4} u_i = \emptyset$$
> (b) Given $u \not\equiv U$ and $d \not\equiv D$, M may be partitioned so that:
> $$\bigcup_{i=1}^{4} m_i = T, \qquad \bigcap_{i=1}^{4} m_i = \emptyset$$
> (c) Given $m \not\equiv M$ and $u \not\equiv U$, D may be partitioned so that:
> $$\bigcup_{i=1}^{4} d_i = T, \qquad \bigcap_{i=1}^{4} d_i = \emptyset$$
> where T represents the total set and $\emptyset$ the empty set.

According to the AICPA's descriptive model of audit opinions an unqualified opinion ideally should be issued when U and M are both kept at a minimum and D at a maximum. Alternatively, when U and M are large or D is small, the auditor should consider issuing a qualified opinion; and when U and M are "significantly" large or D is "unusually" small, the auditor should issue an adverse opinion or disclaim an opinion. This implies that there are certain transitive relations among the alternatives of opinions. On the basis of (a) the AICPA's descriptive model, (b) "Statement 1" above, and (c) the fact that U, M, and D are all

monotone functions, we propose a second basic statement:

Statement 2. Let u_i, m_i, d_i ($i = 1, 2, 3, 4$, each representing one of the four opinions) be subsets of U, M, and D respectively. Then:

(a) Given $m \not\in M$ and $d \not\in D$,
$\{u \not\in u_1\} < \{u \not\in u_2\} < \{u \not\in (u_3 \cup u_4)\}$
(b) Given $u \not\in U$ and $d \not\in D$,
$\{m \not\in m_1\} < \{m \not\in m_2\} < \{m \not\in (m_3 \cup m_4)\}$
(c) Given $u \not\in U$ and $m \not\in M$,
$\{d \not\in d_1\} > \{d \not\in d_2\} > \{d \not\in (d_3 \cup d_4)\}$

Functional Relations Between Parameters

The exact functional relationship of the three parameters to the four alternative audit opinions cannot be determined because of the lack of empirical data available under the measurement scheme proposed in this paper. However, we can intuitively theorize the general relationship by using the descriptive model assumed in the auditing literature. More formal theoretical support will follow once this relationship is understood.

Relationship between opinions and U, M, when D=d. For presentation purposes, only two parameters will be considered at a time, starting with the measures of ignorance (or uncertainty), U, and the monetary effects, M. From various definitions in audit opinions and related discussions, we know that if the effect of an item in question is material ($M > m_1$ in Figure 1) and evidence of it is well substantiated ($U = 0$ in Figure 1), the auditor will choose a qualified opinion. Alternatively, when $M = m_2$ (which is less than m_1) but $U = u_1$ (signifying uncertainty as to the true effect of m_2), then he will still choose a qualified opinion because of the uncertainty associated with m_2. This relationship may be expressed as a negatively sloped line O_1 in Figure 1. A similar argument can be made with respect to O_2, where the intercept on the M-axis (m_3) is normally referred to in the literature as "significantly material." Notice that the line O_3 distinguishes a space between the adverse opinion and

disclaimer of opinion, where the former occupies space representing significant materiality with convincing evidence or low U, and the latter representing significant materiality with high uncertainty or high U. Notice also that all three lines are negatively sloped, depicting the trade-off relations between materiality and uncertainty.

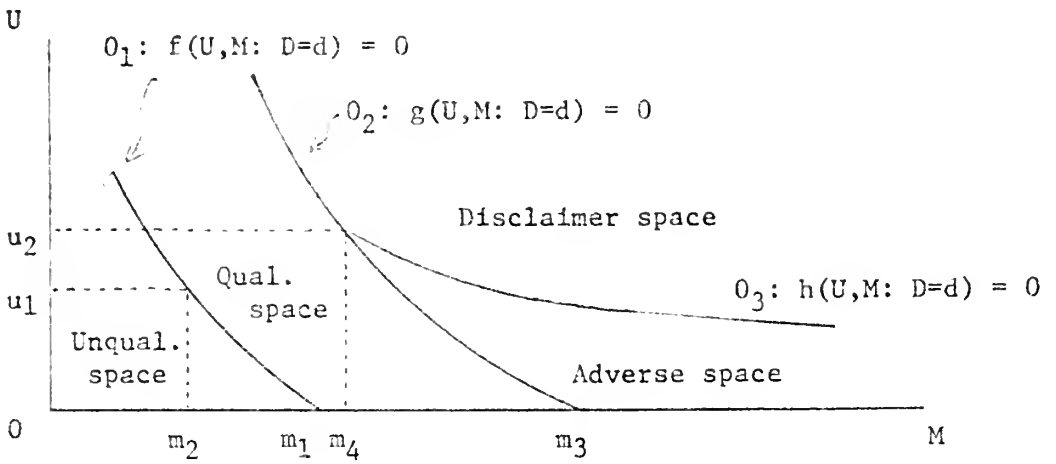

Figure 1

Relationship between uncertainty, materiality, and the audit opinions.

Relationship between opinions and U, D, when M = m_4. In Figure 2, opinion spaces are projected on the U and D dimension space when M = m_4, where m_4 was implied as a material amount in Figure 1. Figure 2 shows us that even in the case of a material item, if U is low (good evidence) and D is large (an exceptionally good disclosure), then an unqualified opinion may be issued. (e.g., a pending litigation where a loss is very definite on the basis of subsequent events information and the case is well disclosed). The figure also shows us that when U and D are both low (low level of uncertainty as to the nature of M but with unsatisfactory disclosure), an adverse opinion is due if M is very large. The other two opinions may be explained similarly. The shape of the curves is chosen a priori because we reason that the marginal effect of disclosure changes first at

an increasing rate; but once a certain level of U is reached, the marginal effect of disclosure changes at a decreasing rate. This is intuitively appealing since, at a high level of uncertainty as to the state of nature, disclosure itself becomes obscure and ineffective.

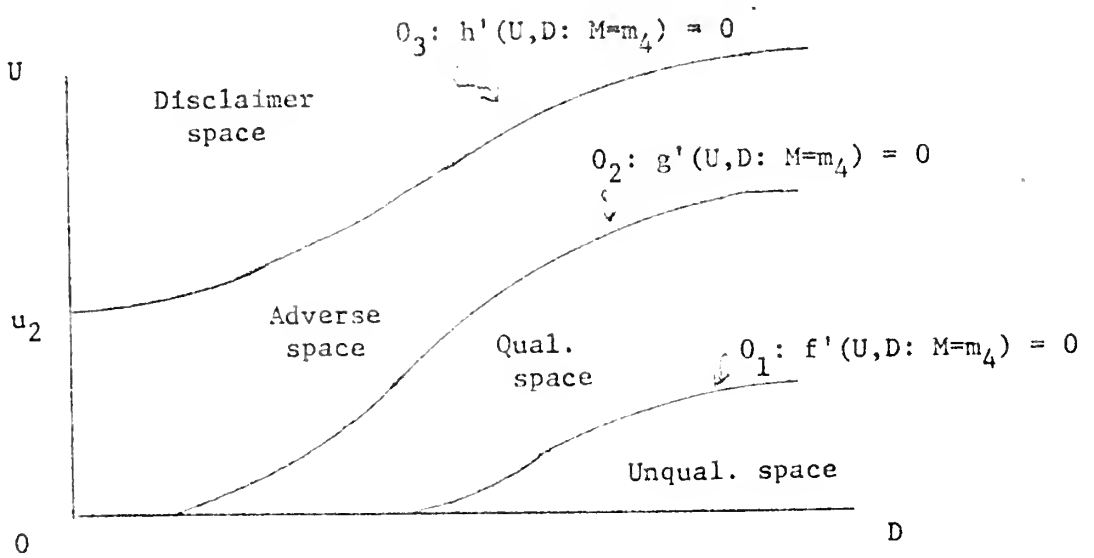

Figure 2

Relationship between uncertainty, disclosure, and the audit opinions

Relationship between opinions and D, M, when U=u. In Figure 3, opinions are projected on the D and M dimension where U = u, and u is assumed to be small. Notice that at small u, or a low uncertainty level, the disclaimer space is not shown because the auditor is not allowed to disclaim when he has definite evidence on which to determine whether or not the financial statements are fairly presented. At a higher level of U, however, the projection will show the disclaimer space instead of the adverse space. The shape of the curves shows that the effect of materiality may be alleviated by improving the level of disclosure.

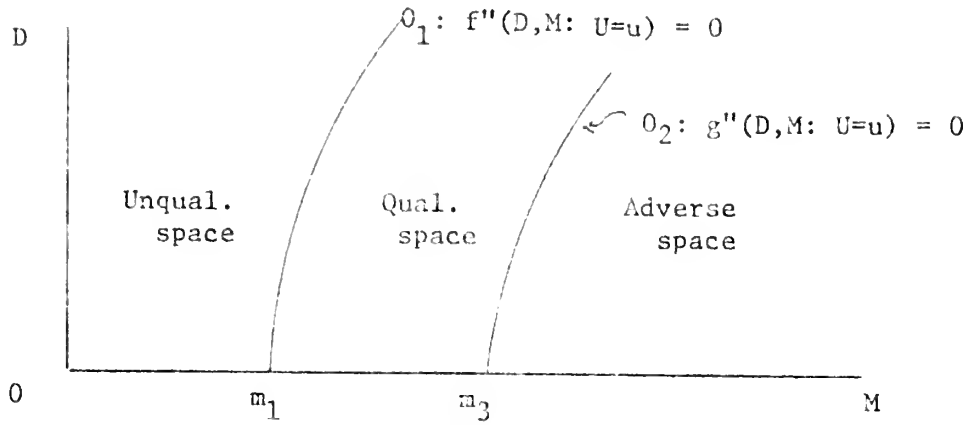

Figure 3

Relationship between disclosure, materiality, and the audit opinions

Partition of Opinion Space

Having investigated some basic concepts underlying audit opinions, we can now make an overall statement concerning audit opinion alternatives and the parameters in terms of the audit opinion vector space.

Suppose there is a three-dimensional vector space generated by basis vectors U, M, and D with the operations of a positive scalar. This space is equivalent to the first octant in a three-dimensional space. Any point in this vector space can be represented by a vector, (u $\in$ U, m $\in$ M, and d $\in$ D). From "Statement 1" and "Statement 2," we can conceive a set of points partitioning between u_i and u_j, (i, j=1, 2, 3, 4 and $i \neq j$). Let this point be called a boundary point and denoted as u'_{ij}. For example, u'_{12} represents a point in U that delineates between the unqualified and the qualified. From "Statement 1" a point u'_{ij} is uniquely determined when m $\in$ M and d $\in$ D are given. Similarly, we can conceive unique boundary points m'_{ij} and d'_{ij} in M and D, respectively. For example, m'_{12} and d'_{12} represent points in M and D, respectively, delineating the unqualified and the qualified. A vector (u'_{12}, m'_{12}, d'_{12}) then is a point in the audit opinion

space delineating the unqualified opinion space and the qualified opinion space. But there are many such vector sets uniquely determined out of the simultaneous interplay of U, M, and D as partially demonstrated in Figures 1, 2, and 3. In fact, there are infinite sets since U, M, and D are all continuous variables. For example, there is an infinite set $\{u'_{12} \in U\}$ determined as D and M change from 0 to ∞; similarly there are infinite sets $\{m'_{12} \in M\}$ and $\{d'_{12} \in D\}$. The vector sets $\{u'_{12}, m'_{12}, d'_{12}\}$ then constitute a continuous surface partitioning between the unqualified opinion space and the qualified. This surface may be functionally expressed as:

$$F\ (U, M, D) = 0 \tag{9}$$

By the same reasoning, we can conceive sets of vectors that uniquely determine surfaces partitioning the qualified opinion space and the adverse; the qualified and the disclaimer, and the adverse and the disclaimer. These surfaces may be defined as:

$$G\ (U, M, D) = 0 \tag{10}$$
$$H\ (U, M, D) = 0 \tag{11}$$
$$K\ (U, M, D) = 0, \tag{12}$$

respectively. But from "Statement 2," there is no surface partitioning the unqualified and the adverse or disclaimer.

Determination of the Boundary Sets $\{u'_{ij}, m'_{ij}, d'_{ij}\}$

Although the conceptual clarification of audit opinion has been made, a formidable problem remains: How do we determine the partitioning surface vector sets $\{u'_{ij}, m'_{ij}, d'_{ij}\}$. The discussion which follows is to show conceptually how such vector sets could be obtained. The discussion provides a theoretical basis for the discussions involving Figures 1, 2, and 3.

Suppose the auditor has obtained specific values for U, M, and D from an actual audit engagement. For the given set of values, the auditor may issue any

one of the four audit alternatives that would bring him the most satisfaction. Conceptually, the satisfaction from issuing a specific audit opinion may be measured in terms of his reputation, the effect on his revenues, the possibility of being sued by others, and the satisfaction to his client and the users of his opinion. In other words, the auditor's satisfaction is his utility which may be expressed as a function of U, M, and D. Assume the auditor's satisfaction from issuing an unqualified opinion is measured as follows:

$$V_1 = V_1 (U, M, D). \tag{13}$$

Given a vector (u, m, d), the auditor will have different satisfactions from issuing a qualified opinion, an adverse opinion, and a disclaimed opinion. Assume the respective satisfactions are measured by equations:

$V_2 = V_2 (U, M, D)$: for issuing a qualified opinion (14)
$V_3 = V_3 (U, M, D)$: for issuing an adverse opinion (15)
$V_4 = V_4 (U, M, D)$: for issuing a disclaimed opinion (16)

To illustrate how the above equation V_i, i=1, 2, 3, 4, is related to the determination of the surfaces denoted by equation (9), (10), (11), and (14), let us turn to Figure 4, where V_i is expressed as a continuous function of a single variable M by holding U and D constant.

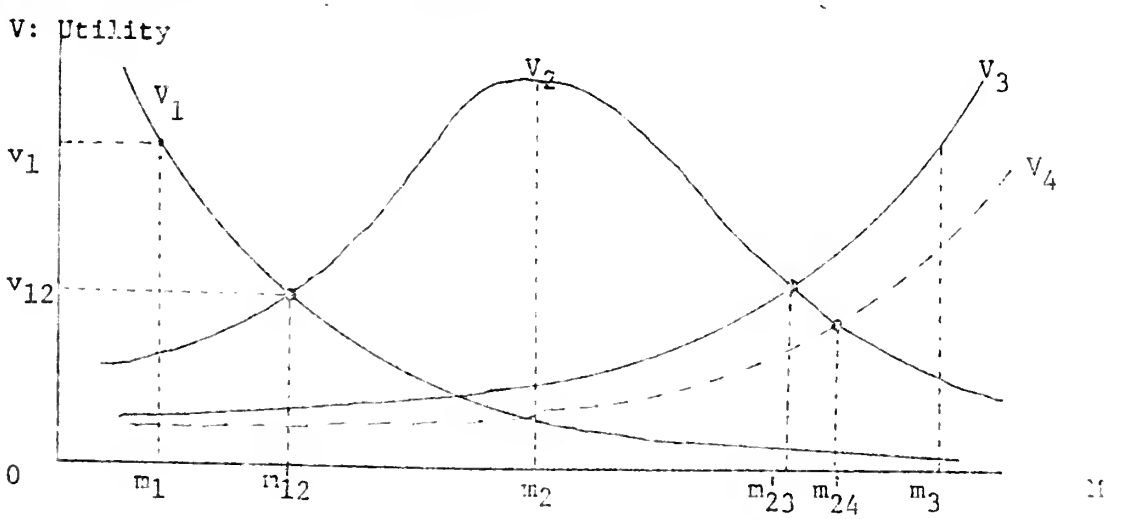

Figure 4

Marginal utility curves corresponding to the four different opinions, with respect to changes of materiality.

The curves show the relationship between M and V_i, which are intuitively appealing: Holding the effects of U and D constant, when M is small such as m_1, the auditor would be most satisfied by issuing an unqualified opinion, but as M becomes large the auditor would be more concerned with the unfavorable consequence of issuing an unqualified opinion, i.e., his satisfaction will be diminishing (See V_1 in Figure 4). On the other hand, as M gets bigger the auditor would be more comfortable by issuing a qualified opinion; once a certain point such as m_2 is reached, however, the magnitude of M becomes so large that he begins to worry and his satisfaction with the qualified opinion will start to diminish (see V_2). When M is small, issuing an adverse opinion or disclaimer of opinion would bring very little satisfaction to the auditor. (In fact, a negative satisfaction is conceivable). But as M gets larger, the adverse opinion or the disclaimer become more justified and eventually at a point where M is very large such as m_3, the satisfaction would be indeed larger than the ones obtained from issuing an unqualified or qualified opinion (see V_3 and V_4). From the shapes of the curves, it is apparent that V_i, i=1, 2, 3, 4, can be interpreted as a marginal utility function showing the change of his satisfaction or utility as M changes. Like any other decision maker, the auditor would choose a course of action which brings in more utility. Therefore, in Figure 4, the auditor will be most satisfied by issuing an unqualified opinion between the interval $(0, m'_{12})$ because $(V_1) > (V_i)$, i=2, 3, 4, in that interval. Similarly he will be most satisfied by issuing a qualified opinion between (m'_{12}, m'_{23}) or (m'_{12}, m'_{24});* by issuing an adverse opinion between (m'_{23}, ∞); or by issuing a disclaimed opinion between (m'_{24}, ∞). The relationship between V_3 and V_4 depends on the other two parameters U and D, particularly U.

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*Whether the interval is (m'_{12}, m'_{23}) or (m'_{12}, m'_{24}) depends on the values of the other two parameters. In any case the boundary point will be the one showing higher utility value.

The point m'_{12} is a partitioning point between the unqualified and qualified, because at the point, $V_1 = V_2$, i.e., the utility is the same. At this point the auditor is <u>indifferent</u> whether he issues the unqualified opinion or qualified opinion. We know from "Statement 1" and "Statement 2" that m'_{12} will be different at different U or D. Then the locus of m'_{12} in a three dimensional space will form the surface partitioning the unqualified space and qualified space. In fact, the curve O_1 in Figure 1 is such a locus of m'_{12} in a two dimensional space where D is held constant. On the curve the auditor is indifferent between issuing an unqualified opinion and a qualified opinion. In this sense the curve O_1 is an

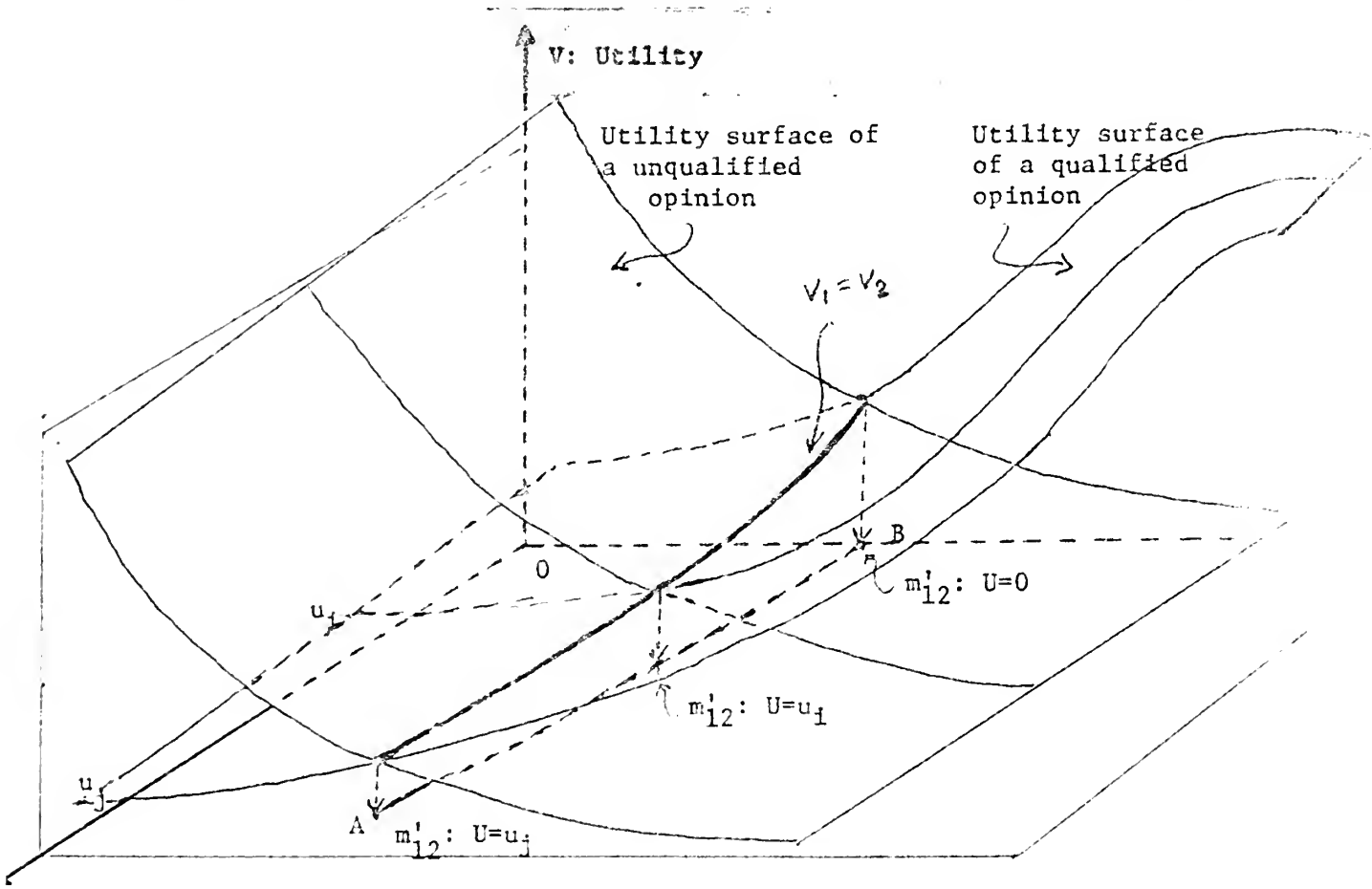

Figure 5

Determination of a line $\overline{AB}$ partitioning between the unqualified and the qualified opinion when uncertainty and materiality factors are used.

"indifference curve."* The line $\overline{AB}$ in Figure 5 shows how the indifference curve O_1 or the locus m'_{12} with respect to the change of U is obtained. Similarly, all other curves in Figures 1, 2, and 3 represent curves showing indifference to respective opinions. Extending to a three dimensional space generated by U, M, and D, we can imagine "indifference surfaces" partitioning opinion spaces.

In sum, the determination of the opinion boundary sets $\{u'_{ij}, m'_{ij}, d'_{ij}\}$ can be made by setting

$$V_i = V_j, \quad i,j = 1, 2, 3, 4, \quad i \neq j. \tag{17}$$

The locus of the set $\{u'_{ij}, m'_{ij}, d'_{ij}\}$ satisfying the condition (17) above is the "indifference surface" partitioning between opinions i and j. Equations (9), (10), (11), and (12) represent such indifference surfaces.

Determination of an Opinion Alternative

Once the space partition is determined by the indifferent surfaces, the process of the audit opinion selection becomes conceptually very much simplified. The auditor must translate the set of significant events E_i, i=1, 2,..., n, into a vector in the audit opinion space and see in which subspace the tip of the vector falls. For example, if the tip falls within the qualified space, then the auditor issues a qualified opinion; if the tip falls within the adverse space, then he issues an adverse opinion--and so forth. By doing so, the auditor is assured to maximize his utility.

Conclusion

This paper demonstrates that the audit opinions can be mathematically

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*The term "indifference curve" should not be confused with the term used in the micro-economic theory, where the concept of indifference is used with respect to utility. In this paper the term indifference is used with respect to the two alternative opinions.

modeled. Undoubtedly, the ideas expressed need further development and sophistication. From the model, however, we observe a few interesting characteristics, as compared to the conventional approach, regarding some aspects of the audit opinion decision process. First, the model recognizes the multiple attributes of event E_i and expresses it in three dimensional terms. Each represents a specific dimension. This allows the auditor to systematically translate the effect of various events E_i, i=1, 2,..., n, into common denominators, expressed in the form of a vector. The auditor determines his audit opinion on the basis of this vector as opposed to the conventional approach in which each effect of the individual events on the opinion is more or less separately taken.

Second, the use of the model does not preclude the use of the auditor's judgment. Judgment is indispensable in auditing; but the use of judgment under the model is directed to individual items such as the determination of $p(E_i)$, $r(E_i)$, $q(E_i)$, e_i, and V_i. This means that the judgment tends to be more reliable and less erratic as the auditor deals with more specific, isolated items. A more complex judgment such as the interrelations of the individual items and their consequences are aided by the model.

A third interesting derivative that is observed from the model is that the meaning of materiality in auditing may possibly be subject to a new interpretation. A well accepted definition of materiality says that "an item should be regarded as material if there is reason to believe that knowledge of it would influence the decisions of an informed investor"[1, p.8]. The definition emphasizes the importance of the investor's interest, and the auditor is forced to take the hypothetical position of an informed investor in deciding whether an item is material. Under this definition, an auditor as a decision maker does not

formally take into account his own utility function. This approach seems somewhat unrealistic, unless we accept a hypothesis that auditors are altruistic public servants, ignoring their own utilities.

Under the model described in this paper, however, the threshold of materiality is determined at the point where the auditor's utility from issuing an unqualified opinion is the same as the utility from issuing a qualified opinion. This does not mean that under the model the auditor ignores the investor's interest in his decision making. The investor's interest is indirectly reflected in the auditor's utility function, as the society provides a means to regulate irresponsible auditors. If the auditor wants to meet the responsibility entrusted upon him from the society, he has to reflect the investor's interest in his own utility function. The materiality concept under the model thus shifts the emphasis from the auditor taking the hypothetical position of an investor to the one reflecting his own utility in which the investor's interest is integrated as part of the auditor's overall utility.

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